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NEAR FIELD STATIC TILT FROM SURFACE LOADS.(U)  
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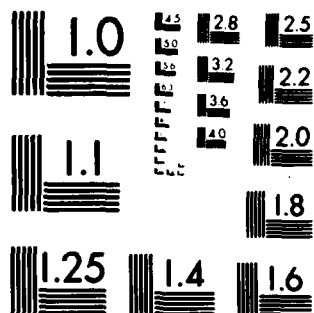
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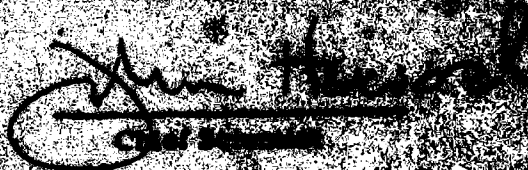
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## Preface

The authors should like to acknowledge several interesting and helpful discussions with Mr. J. Battis. They also thank Mr. H. A. Ossing for supplying the dispersion data.

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## Near Field Static Tilt From Surface Loads

### 1. INTRODUCTION

Tiltmeters are usually placed on horizontal surfaces (a mine tunnel floor) or attached to vertical surfaces (the wall of a vertical borehole), measuring the tilt of initially horizontal and vertical line elements, respectively. While the tilt of a horizontal and of a vertical surface is not the same quantity, at the free surface of an elastic body they become equal (Appendix A). Historically, tilt measurements have been made in mine tunnels to avoid contamination of the data by meteorological effects near the surface. These instruments were practically always placed on a horizontal platform, but because it was assumed that they were near enough to the free surface, horizontal and vertical tilts were considered equal. Therefore, no confusion arose when comparing theoretical tilt with measured tilt. There are occasions, however, when care must be taken in making the foregoing comparison. This report discusses the theoretical calculation of tilt due to a stationary vehicle and compares results with the observed tilt measured in the field.

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(Received for publication 8 July 1980)

## 2. FIELD EXPERIMENT

The purpose of the experiment was to ascertain if under field conditions a tiltmeter or an array of tiltmeters could determine the vehicle load distributions.

The experiment included the installation of a single tiltmeter at a shallow depth (~6 ft). The installation procedure was essentially the one used by the U.S. Geological Survey.<sup>1</sup> Briefly, the installation involved augering a vertical hole into the alluvium and placing the tiltmeter, which is housed in a 48-inch long capsule, into the augered hole. A mixture of dry sand was then compacted around the tiltmeter case.

The top of the capsule was approximately 2 ft below the ground surface. After completion of the installation, the tiltmeter was allowed to operate for a sufficient length of time to stabilize.

The experiment involved driving a vehicle (standard sedan) of known weight up to and away from the tiltmeter on a line parallel to one of the sensitive axes of the instrument. The vehicle stopped at predetermined distances from the tiltmeter and the tilt was recorded. The vehicle then moved to the next position, and so on.

The data showed that the derivative of the measured tilt underwent a sign reversal, as can be seen from Figure 1.

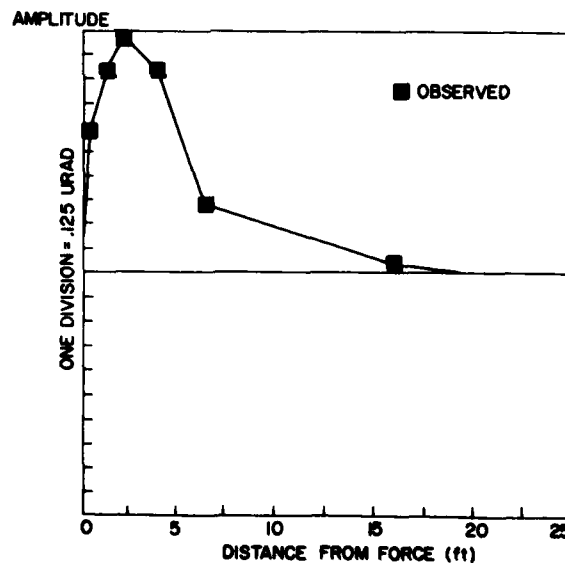


Figure 1. Experimental Tilt Data

1. Johnston, M.J.S., and Mortensen, C.E. (1974) Tilt precursors before earthquakes on the San Andreas Fault, California, Science 186:1031-1034.

### 3. COMPUTATION OF TILTS

The usual computation of the horizontal tilt of a free surface, for an infinitesimal line segment, is not able to explain the sign reversal tilt observed in the field data. But, the tiltmeter used in this experiment is not in fact measuring the horizontal tilt of the free surface, but rather the vertical tilt at depth. Furthermore, the tilt sensed by the tiltmeter (a 4-ft capsule) is approximately the tilt of a finite vertical line element, which is not the same as the tilt of an infinitesimal line element, in an inhomogeneous strain field.

Based on the foregoing considerations, we now calculate the vertical tilts sensed by a tiltmeter emplaced in a finite capsule.

### 4. MODEL

We make the following two initial assumptions:

1. The soil responds elastically to the loading.
2. The capsule behaves as a rigid body compared to the soil, and can be treated as a finite length line element of negligible width.

The first assumption appears justified due to the fact that the tiltmeter returned to its initial position when the load was removed.

The second assumption is based on a comparison of Young's Moduli ( $E$ ), of steel and soil, approximately  $3.0 \times 10^7$  lb/ft<sup>2</sup> and  $4.32 \times 10^6$  lb/ft<sup>2</sup>, respectively, and the length to width ratio (32:1) for the capsule.

The elastic constants chosen for the model are based on compressional and shear wave velocities derived from an empirical dispersion curve for the local area.<sup>2</sup> Using reasonable values for soil densities, we found Young's modulus to be  $4.32 \times 10^6$  lb/ft<sup>2</sup> and Poisson's ratio ( $\delta$ ) was chosen to be 0.25.

Based on the geometry shown in Figure 2a, we calculated the vertical tilt at depth of the capsule as a function of distance from the vehicle. To gain insight into the problem, we approximated the four loads of the vehicle's wheels as a single point force, the vehicle's weight,  $3.2 \times 10^3$  lb. The more realistic calculation which computes the sum of the four tilts caused by the loads of the vehicle's wheels is presented later.

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2. Ossing, H. A. (1979) Personal communication.

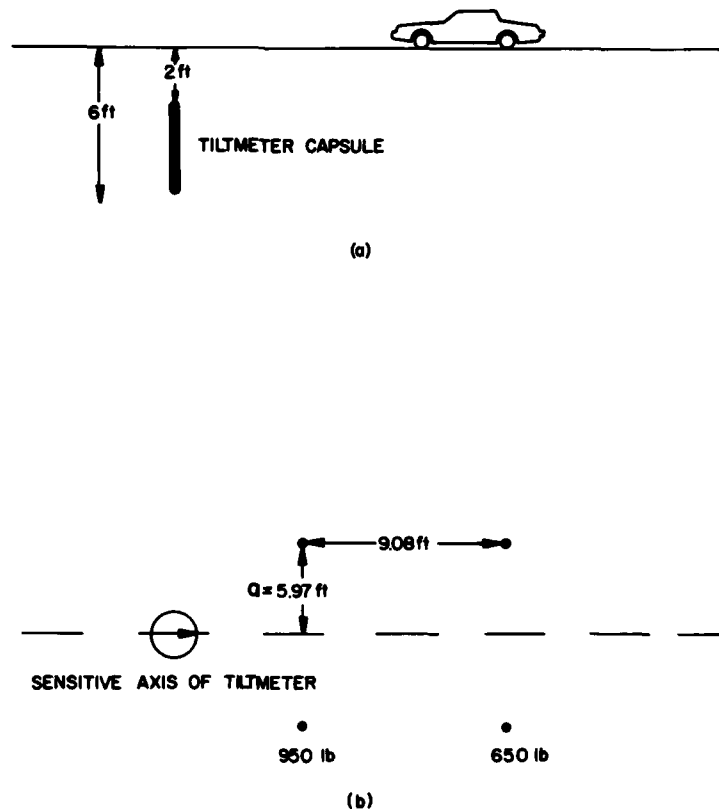


Figure 2. Experimental Configurations

## 5. EXPRESSIONS FOR TILTS

Farrell<sup>3</sup> gives expressions for the vertical and horizontal displacements at depth due to a single unit point force on the surface of an elastic half space. Farrell uses a cylindrical coordinate system with basis vectors  $e_z$ ,  $e_r$ , and  $e_\theta$  and lets  $z \leq 0$  be the volume occupied by the half space. The problem is axially symmetric, so that there is no  $\theta$  dependence in the solution.

3. Farrell, W. E. (1972) Deformation of the earth by surface loads, Rev. Geophys. and Space Phys. 10(No. 3):767.

$$\begin{aligned}
 u(z, r) &= -\frac{1}{4\pi\mu R} \left( \frac{\sigma}{\eta} + \frac{z^2}{R^2} \right) \\
 v(z, r) &= -\frac{1}{4\pi\eta r} \left( 1 + \frac{z}{R} + \frac{\eta r^2 z}{\mu R^3} \right)
 \end{aligned} \tag{1}$$

where

$$\sigma = \lambda + 2\mu$$

$$\eta = \lambda + \mu$$

$$R^2 = r^2 + z^2$$

with  $\lambda$  and  $\mu$  being Lamé parameters.

Using the same coordinate system, we define the horizontal and vertical tilt of an infinitesimal line element, respectively, as

$$\begin{aligned}
 \lambda_H &= \frac{\partial u}{\partial r} \\
 \lambda_V &= \frac{\partial v}{\partial z} .
 \end{aligned} \tag{2}$$

The quantities  $u$  and  $v$  are the vertical and horizontal displacements, respectively; that is, the tilt  $\lambda_H$  is equal to the change in the vertical displacement between two points, as that distance approaches zero, and similarly for  $\lambda_V$ .

At the surface, these two tilts become equal in magnitude but, opposite in sign, though the same is not true at depth (Appendix A).

Performing the operations indicated by the defining equations for the tilt shown above on Farrell's Eq. (1), we obtain the expressions for the horizontal and vertical tilts of an infinitesimal line element.

$$\begin{aligned}
 \lambda_H &= \frac{Pr}{4\pi\mu} \left[ \frac{\sigma}{\eta} (r^2 + z^2)^{-1.5} + 3z^2 (r^2 + z^2)^{-2.5} \right] \\
 \lambda_V &= \frac{-P}{4\pi\eta r} \left[ (r^2 + z^2)^{-0.5} + (r^2 + z^2)^{-1.5} \frac{\eta r^2}{\mu} \right. \\
 &\quad \left. - (r^2 + z^2)^{-1.5} z^2 - \frac{3\eta r^2 z^2}{\mu} (r^2 + z^2)^{-2.5} \right]
 \end{aligned} \tag{3}$$

where  $P$  = force.

For finite line elements,  $\lambda_V$  can be approximated by

$$\lambda_V \approx \bar{\lambda}_V = \frac{\Delta v}{\Delta z} = \frac{v_2 - v_1}{z_2 - z_1} \quad (4)$$

where the subscripts refer to the extremities of the capsule. Subscript 2 designates the deeper of the two extremities; subscript 1, the shallower; and  $z$  represents the depth below the surface;  $v_1$  and  $v_2$  are the horizontal displacements of the shallow and deep extremities of the capsule. In the present problem  $z_2$  and  $z_1$  are -6 ft and -2 ft, respectively; therefore Eq. (4) becomes

$$\bar{\lambda}_V = \frac{\Delta v}{\Delta z} = \frac{v_2 - v_1}{-4.0 \text{ ft}}.$$

## 6. THEORETICAL RESULTS

We first examine the vertical and horizontal displacements at a depth of -4 ft due to a point force (see Figure 3). Qualitatively, the vertical displacement is negative infinity at  $r = 0$ ; it approaches zero as  $r$  approaches infinity, but never becomes positive. The horizontal displacement also is negative near the load; it goes through zero and becomes positive approximately 8 ft from the force. A positive horizontal displacement represents a displacement away from the force in the positive  $r$  direction.

We next examine the analytic horizontal and vertical tilts (Appendix A) shown in Figure 4. A positive tilt is a clockwise rotation in the region of positive  $r$ . Note that the horizontal tilt has a turning point at  $\sim 4.0$  ft, approaches zero but never becomes positive. The vertical tilt also has a turning point at  $\sim 4.0$  ft, then crosses zero and becomes positive.

In Figure 5, the vertical numerical (Appendix A) pipe tilt, at a depth of -4.0 ft is plotted with the analytic horizontal and vertical tilts. Clearly the capsule and vertical tilt have the same qualitative signature, but differ in magnitude. They also cross zero at different distances.

Figure 6 shows the relationship between the horizontal displacements at a depth of -2 ft and -6 ft, and the vertical capsule tilt. When  $v_2 = v_1$  [Eq. (4)], the capsule tilt is zero as shown at approximately  $r_0 = 4.0$  ft. Now note the part of the figure to the left of the point where  $v_2 = v_1$  ( $r = r_0$ ). For values of  $r < r_0$ , we see that  $v_1$  is always more negative than  $v_2$ . This means that the top extremity of the capsule is displaced closer to the origin than the bottom extremity, resulting in a negative tilt. For values of  $r > r_0$ , the opposite is true and the capsule experiences a positive tilt.

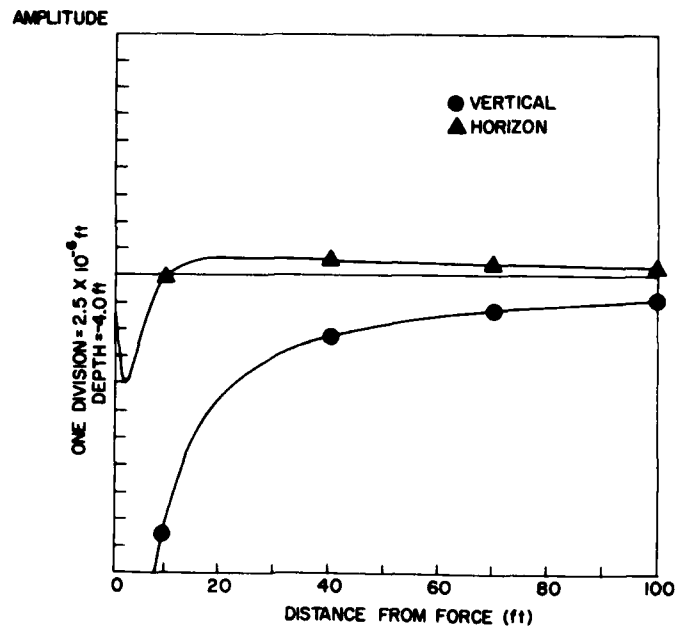


Figure 3. Displacements Due to a Point Force.  
(Depth = -4.0 ft,  $E = 4.32 \times 10^6$  lb/ft<sup>2</sup>,  
 $\delta = 0.25$ ,  $P = 3.2 \times 10^3$  lb)

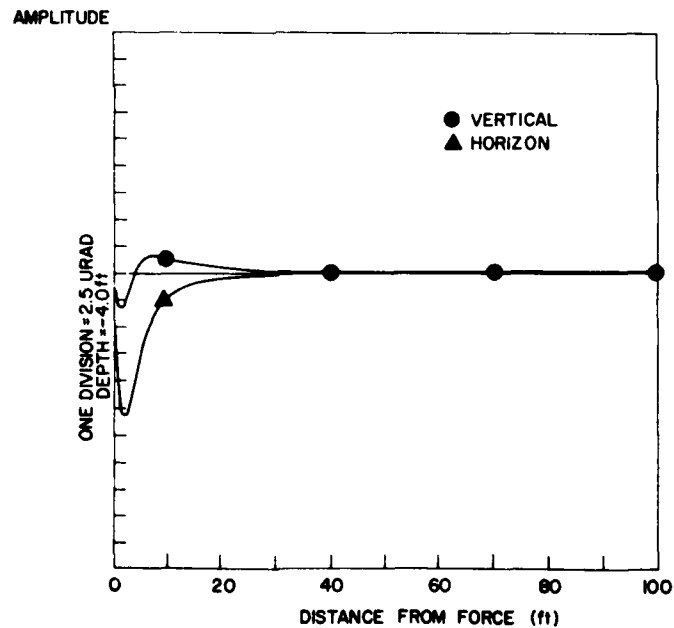


Figure 4. Analytic Tilts Due to a Point Force.  
(Depth = -4.0 ft,  $E = 4.32 \times 10^6$  lb/ft<sup>2</sup>,  
 $\delta = 0.25$ ,  $P = 3.2 \times 10^3$  lb)

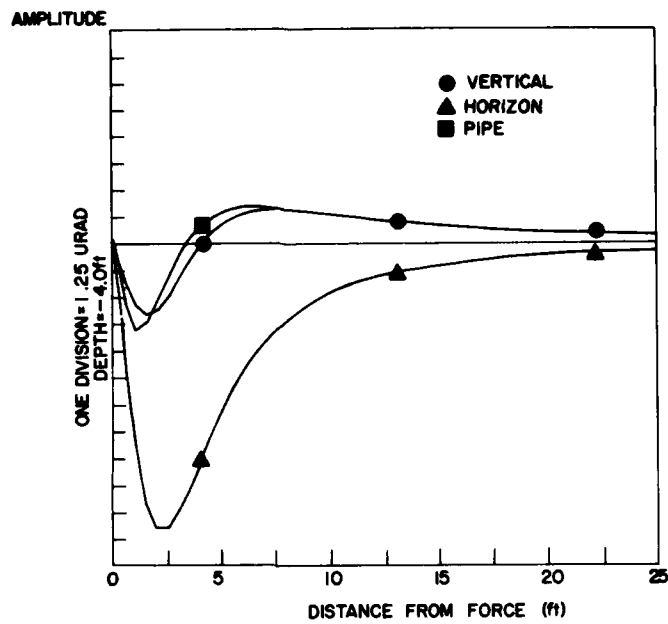


Figure 5. Analytic Tilts and Pipe Tilt Due to a Point Force. (Depth = -4.0 ft,  $E = 4.32 \times 10^6$  lb/ft<sup>2</sup>,  $\delta = 0.25$ ,  $P = 3.2 \times 10^3$  lb)

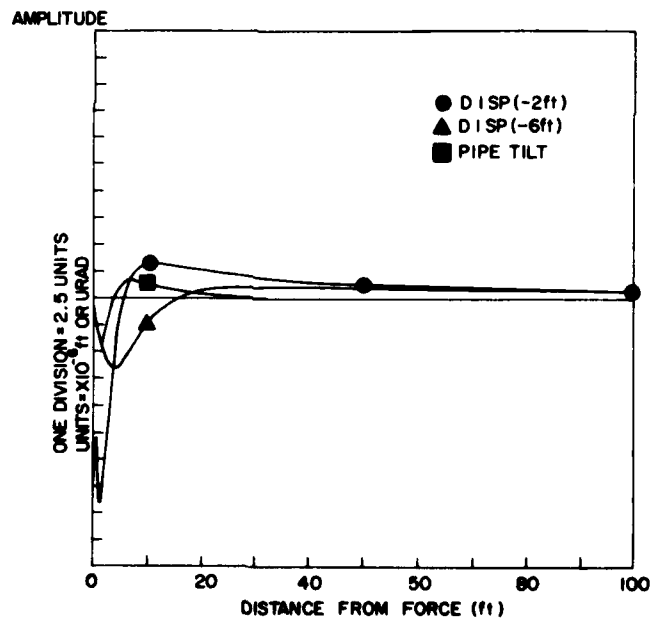


Figure 6. Pipe Tilt and Displacements Due to a Point Force. (Depths of horizontal displacements are at -2.0 and -4.0 ft,  $E = 4.32 \times 10^6$  lb/ft<sup>2</sup>,  $\delta = 0.25$ ,  $P = 3.2 \times 10^3$  lb)



Figure 7 shows the difference in the capsule tilt when 4 point forces, representing the vehicle's wheels are contrasted with a single point force. In both cases the total force is 3200 lb.

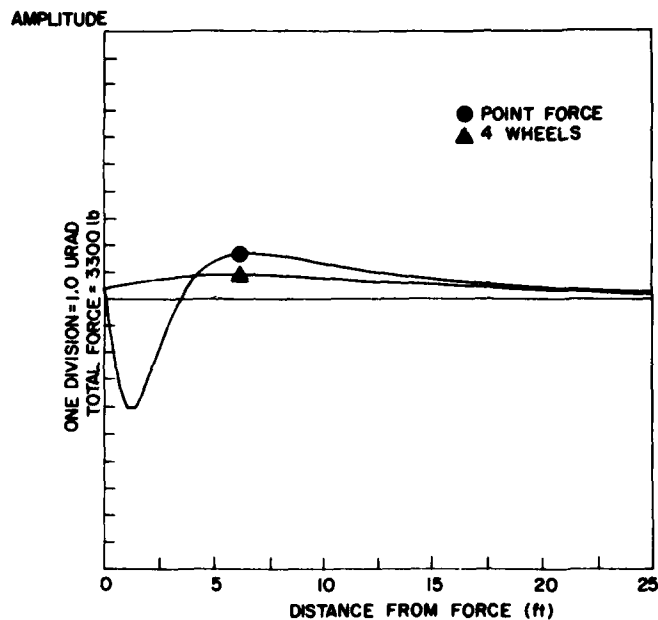


Figure 7. Analytic Tilts Due to a Point Force and Four Point Forces. (Depth = -4.0 ft,  $E = 4.32 \times 10^6$  lb/ft<sup>2</sup>,  $\delta = 0.25$ ,  $P = 3.2 \times 10^3$  lb for both one and four point forces)

The tilt due to 4 point forces is the sum of the projections of the tilts due to each wheel along the sensitive axis of the tiltmeter. The force used for each of the front wheels was 950 lb and for each of the rear wheels 650 lb. Actually, the distance from the force is the distance to the midpoint of the front axle in the case of the 4 point forces; otherwise, it is simply the distance to the point force. The geometry due to the 4 wheels is shown in Figure 2b with the dimensions used in the computations. The tilt due to 4 point forces shows only one turning point and qualitatively does not even resemble the tilt due to a single force, except at large distances ( $r \geq 20$  ft). At small  $r$  the tilt due to the 4 point forces is not zero, as the rear wheels still contribute (recall that distance is measured to the front axle). Figure 8 shows the separate contributions of the 2 front and the 2 rear wheels to the total tilt.

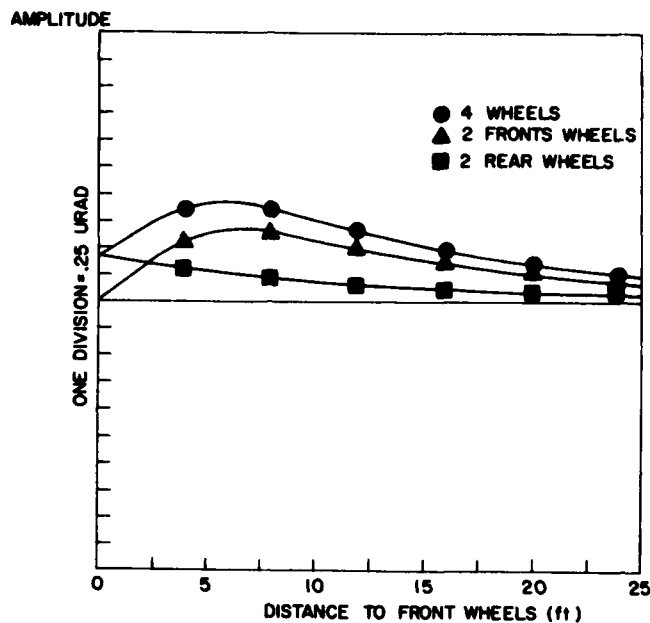


Figure 8. Pipe Tilt Due to Two Front and Rear Wheels.  
(Depth -4.0 ft,  $E = 4.32 \times 10^6$  lb/ft<sup>2</sup>,  $\delta = 0.25$ ,  
 $P = 3.2 \times 10^3$  lb)

## 7. MODEL VS EXPERIMENT

If we now halve the distance between the wheels of the vehicle (the quantity  $a$ , shown in Figure 2), we approximate the dimensions of the vehicle used in the field experiment. Figure 9 shows the computed pipe tilt and the tilt measured in the field. Comparison of the pipe tilt shown in Figures 7 and 9 shows that as the point forces move closer to each other, the pipe tilt more closely resembles the tilt due to a single point force. Comparison with the field data is qualitatively good, but the peaks of the curves do not match well.

Varying the elastic constants has a negligible effect on the position of the peak of the computed tilt curve.

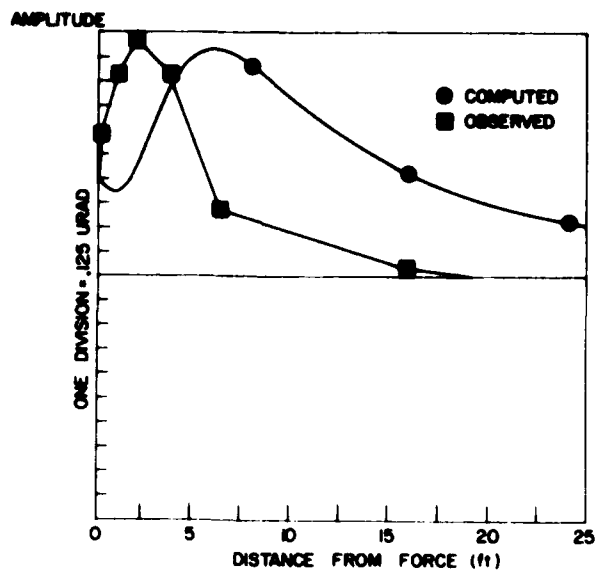


Figure 9. Pipe Tilt Due to Four About Equally Spaced Point Forces and the Experimental Tilt. (Depth = -4.0 ft,  $E = 4.32 \times 10^6$  lb/ft<sup>2</sup>,  $\delta = 0.25$ ,  $P = 3.2 \times 10^3$  lb)

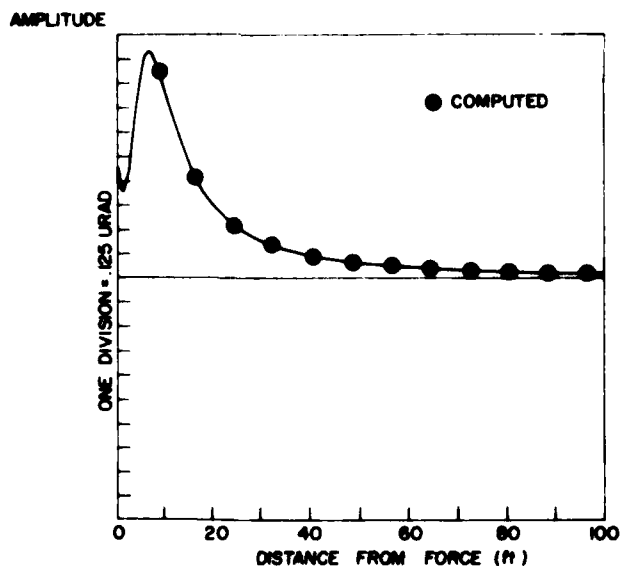


Figure 10. Pipe Tilt Due to Four About Equally Spaced Point Forces. (Pipe tilt on a smaller distance scale)

We then attempted to simulate the influence of the stiff electrical cable at the top of the tiltmeter. We increased the length of the capsule to 6 ft, letting the top be at the ground surface. These changes moved the peak to 3 ft (the experimental peak is at approximately 2.5 ft). Also, increasing Young's modulus to 3 times its estimated value ( $4.32 \times 10^6$  to  $1.296 \times 10^7$  lb/ft<sup>2</sup>) makes the amplitude of the computed curve agree well with the experimental curve. The resulting tilt from both of these changes can be seen in Figure 11.

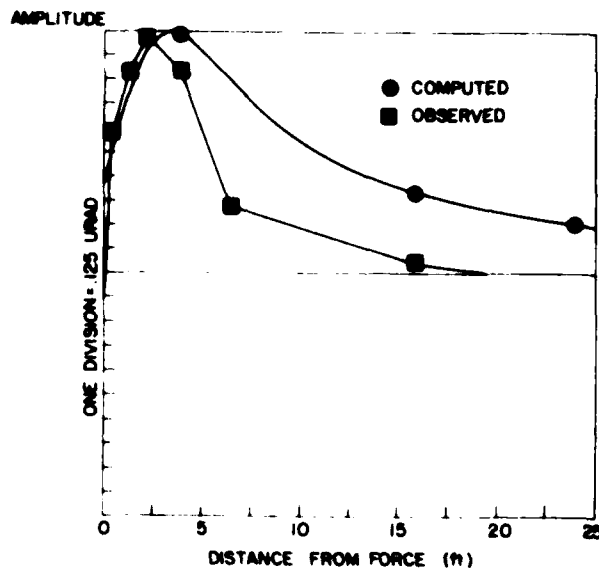


Figure 11. Comparison of Pipe Tilt and Experimental Data (top of tiltmeter is at surface of ground,  $E = 1.296 \times 10^7$  lb/ft<sup>2</sup>,  $\delta = 0.25$ ,  $P = 3.2 \times 10^3$  lb)

## 8. DISCUSSION

We have shown that the correct tilt to compute is the numerical vertical tilt, at depth, experienced by a finite length (4 ft) capsule. We approximated the loads of the vehicle's 4 wheels as point forces and computed the total tilt due to these forces. This result is shown in Figure 11, along with a plot of the experimental data.

At the present time, there appear to be five likely sources of error that could contribute to the discrepancy between the computed tilt and measured tilt. These

are errors in the estimation of the elastic parameters of the soil, possible errors introduced by the 4 to 6 inches of asphalt at the ground surface, that is, not considering a layered half space, the sand-filled cavity into which the tiltmeter capsule was installed, a "stiff" electrical cable connected to the tiltmeter, the approximation of the wheel loads as point forces, and finally geologic effects. We based our estimates of the elastic constants on reasonable values, in agreement with standard references,<sup>4</sup> but no attempt was made to measure these at the field site. The possible errors introduced by approximating a 2-layer medium (4 in. to 6 in. of asphalt over a soil half space) should be small, as the effect of surface layers dies off as a function of layer thickness. In this problem, the thickness of the asphalt is small compared to the scale of the problem. The sand filled cavity may introduce errors that are quite difficult to estimate. The most practical approach would be to model the situation utilizing the finite element method, and the same approach is suggested for estimating more realistically the effects of a stiff cable, and also geologic inhomogeneities. The expressions for the displacements due to a point force and a force uniformly distributed over a circle of radius  $b$  are equivalent at distances of approximately  $r \geq 5b$ . It therefore seems reasonable to use the expressions for the point force in this study.

## 9. CONCLUSION

We have shown that a reasonable approximation to the observed tilt can be made using a simple linear isotropic elastic model. This preliminary work suggests that the finite element method be utilized in any further study, in order to model the physical problem more accurately. The study could also be extended to include a moving vehicle, which is the more probable field situation. Also, Figure 4 shows that monitoring horizontal tilt should increase the amplitude of the measured tilt signal by approximately 4.

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4. Lambe, T. W., and Whitman, R. W. (1969) Soil Mechanics, Wiley & Sons, New York, New York.

## Appendix A

### Tiltmeter Measurement in the Field

In this section we discuss briefly the quantity that a tiltmeter measures in the field. First, tiltmeters are usually coupled either to horizontal or to vertical surfaces. It is also assumed that the installation is close enough to the free surface so that the free surface boundary conditions are applicable, namely, that the shear stress is zero; hence the shear strain vanishes.

We now consider a right-handed Cartesian coordinate system and define vertical and horizontal tilts about the  $x_2$  axis to be, respectively,

$$\begin{aligned}\lambda_V &= \frac{\partial u_1}{\partial x_3} \\ \lambda_H &= \frac{\partial u_3}{\partial x_1}\end{aligned}\tag{A1}$$

where  $u_i$  is the displacement field.

We disregard rotations about the  $x_1$  and  $x_3$  axes for simplicity. The vertical and horizontal tilts can be interpreted as rotations of lines initially vertical and horizontal before deformation. While in general, these two quantities are not equal, for the special case when the shear strain is zero, they become equal in magnitude and opposite in sign. Thus at the free surface

$$\epsilon_{13} = \frac{1}{2} \left( \frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right) = 0 \quad (A2)$$

and algebra yields

$$\frac{\partial u_1}{\partial x_3} = - \frac{\partial u_3}{\partial x_1} \quad (A3)$$

This shows that it makes no difference whether one monitors vertical or horizontal tilt at the free surface; they are equivalent.

The foregoing discussion refers to infinitesimal line segments. If the strain field is uniform, the above equations for tilt would be applicable, even if we employed a finite length tiltmeter. Because the tiltmeter we are dealing with is not infinitesimal in length, and the strain is not uniform close to the source, the Eqs. (A2) and (A3) must be written in the following form:

$$\lambda_V = \frac{\Delta u_1}{\Delta x_3} = \frac{u_1^2 - u_1^1}{\Delta x_3} \quad (A4)$$

$$\lambda_H = \frac{\Delta u_3}{\Delta x_1} = \frac{u_3^2 - u_3^1}{\Delta x_1}$$

where the subscripts refer to the coordinate axes directions, the superscripts refer to the extremities of the capsule, and  $\Delta x_i$  is a finite length, in this case the length of the capsule.

Equations (A1) and (A4) become equal as  $\Delta x_i \rightarrow 0$  and as the distance from the source at which the tilt is calculated approaches infinity (the distance from the source at which the strain becomes uniform). Equations (A1) and (A4) are designated the analytic and numerical tilt, respectively.

Care must be taken to use the appropriate expression for the displacement in Eqs. (A1) and (A4), depending on whether or not the tiltmeter is located at the free surface or below the free surface.

In the present study we have used the equations for displacement and tilt at depth as, strictly speaking, the tiltmeter is not located at the free surface.